Standardisation in Rewriting – ISR 2014

Assorted exercises

August 28, 2014

1. Determine whether the following derivations is $\prec_l$ and $\prec_o$-standard.

$$d : f(\underline{a}, a) \to f(b, \underline{a}) \to \underline{f}(b, \underline{b}) \to g(b)$$

from the TRS:

$$\begin{array}{rcl} f(x, b) & \to & g(x) \\ a & \to & b \end{array}$$

2. Is the composition of standard derivations, standard? Justify your answer.

3. Consider the following function LMC

$$\begin{array}{rcl} LMC(u) & \stackrel{\text{def}}{=} & u \\ LMC(u; d) & \stackrel{\text{def}}{=} & \begin{cases} v & \exists v.v \prec_l u \text{ and } v[\![u]\!]LMC(d) \\ u & \text{otherwise} \end{cases} \end{array}$$

Define the algorithm:

$$\begin{array}{ll} u_1 = LMC(d) & d_2 = d[\![u_1]\!] \\ u_2 = LMC(d_2) & d_3 = d_2[\![u_2]\!] \\ u_3 = LMC(d_3) & d_4 = d_3[\![u_3]\!] \\ \ldots & \\ \quad \text{Stop if } d_i = \mathsf{id}_M & \end{array}$$

Apply it to the derivation d.

4. Consider the algorithm of Exercise 3 where LMC is replaced by:

$$\begin{array}{rcl} OMC(u) & \stackrel{\text{def}}{=} & u \\ OMC(u; d) & \stackrel{\text{def}}{=} & \begin{cases} v & \exists v.v \prec_o u \text{ and } v[\![u]\!]OMC(d) \\ u & \text{otherwise} \end{cases} \end{array}$$

Apply it to d. What can you observe?

5. Consider the algorithm of Exercise 3 where LMC is replaced by:

$$\begin{array}{rcl} EXT(u) & \stackrel{\text{def}}{=} & u \\ EXT(u;d) & \stackrel{\text{def}}{=} & \left\{ \begin{array}{ll} v & \exists v.u \not\leq_o v \text{ and } v[\![u]\!]EXT(d) \\ u & \text{otherwise} \end{array} \right. \end{array}$$

   Apply it to d.

6. Apply the last algorithm to the derivation:

$$d : f(\underline{a}, a) \to f(b, \underline{a}) \to f(b, \underline{b}) \to f(\underline{b}, c) \to f(c, c)$$

   from the TRS:

$$\begin{array}{rcl} a & \to & b \\ b & \to & c \end{array}$$

   Exhibit another standard derivation for d different from the one returned by the algorithm.

7. Consider the setting of the λ-calculus. Show that the algorithm of Exercise 3 is terminating. Hint: Assume that d is the shortest derivation on which it runs indefinitely. Consider $u; d$. Apply the algorithm and deduce the result by contradiction with finite developments.